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# AN INVESTIGATION OF THE INDEPENDENCE OF TIME STUDY ELEMENTS

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## ABSTRACT

The purpose of this investigation was to make an exploratory study of the relationship among the elements of a work cycle to determine if they were statistically independent. The assumption of element independence has been questioned practically since the inception of time study.

The basic data for two operators, one stable and one unstable, were obtained from a micromotion study of a short cycle operation being performed in an industrial plant. Eight film shots, taken at approximately one hour intervals, were available for each operator. Work cycles containing major departures from the usual work method were eliminated. A five and two element breakdown was made on the data for each operator by shots. These data were then subjected to Wilks' multivariate test of independence in order to test the following null hypothesis: the elements of a cycle are independent.

It was found that there was evidence of correlation among the elements of the work cycle for both the five and two element breakdowns. In addition, the following conclusions were drawn on the basis of the test results:

1. There was an indication that the degree of correlation among the elements of a cycle does not remain constant for the same operator during the work shift.

2. The nature and extent of correlation among the elements of a work cycle from period to period appeared to depend on the operator.

3. It appeared that in those shots where the degree of correlation was found to be the highest, that there was a concentration of variables.

4. There was an indication that the degree of correlation among the elements of the five element breakdown was decreased by combining these elements into a two element breakdown of the operation.

5. The grouping process did not decrease the degree of correlation among the elements to the same extent for the same operator on data for different shots.

6. The stable and unstable operators exhibited similar characteristics in regard to the degree and extent of correlation among the elements of the cycle.

It was recommended that a further analysis be made using the basic data from this investigation to determine the coefficient of correlation among the elements from shot to shot. In conjunction with this study it was recommended that the film be re-analyzed and all variables classified by elements by shots. Such a study would provide information concerning the effect of the variables on correlation and the work patterns of the operators. Further, it is recommended that the data for the original twelve element breakdown be treated in a similar manner and compared with the five and two element breakdown.

# CHAPTER I

# INTRODUCTION

## Introduction and History of Time Study

Origin and objectives.--The practice of time study was originated by Frederick W. Taylor at the Midvale Steel Company in 1881 to combat what he termed "soldiering," which was necessitated by rate cutting under the piecework system then in effect (9, pp. 37-38). Taylor intended to objectively and scientifically solve the rate-setting problem through the use of time study. The establishment of time study also represented the first phase in the development of an idealized form of management, "Scientific Management," which was later proposed by Taylor as a substitute for the irrational, subjective, and unsystematic management methods then employed (9, p. 37).

Time study methodology and techniques.--In originating time study, Taylor formulated crude principles and established techniques and methods on which the practice was based. As time study spread Taylor's objective approach was not retained and it became more systematic and mechanistic (9, p. 40). Taylor's techniques were accepted as authoritative principles and no objective studies of the fundamental bases of time study were attempted (9, p. 40).

Although the question of whether or not time study could be considered scientific has been the source of much controversy practically since inception, the original concepts advanced by Taylor have remained basically unchanged until recent years. The changes that were made represented only refinements and modifications to the basic techniques and methods. A critical analysis of prevalent practices in the field led Presgrave to conclude in 1945 that time study was in a "most unsatisfactory state," and that methods had been little improved in precision, manner, or uniformity (27, p. 15).

In 1948, Gomberg (15) suggested a statistical approach to time study. Since that time there has been a movement to implement the application of statistical methods and techniques to time study. Various approaches have been suggested by Sylvester (32), Davidson (9), Abruzzi (1), Lehrer (20), Wilkinson (35), Desmond (11), and others. This move toward a statistical frame of reference was made in an effort to establish scientifically valid time study techniques and methods.

General areas of controversy concerning time study.--The controversy concerning time study seems to center about two general areas; first, the assumption that time study elements are independent, and second, the subjective leveling and rating techniques presently employed.

## The Elemental Time Study Technique

Basic steps as proposed by Taylor.--One of the first steps of Taylor's system of time study was to break the operation down into a number of sub-tasks or elements. These elements were then timed with a stop watch and the time for each recorded. From these, standard elements were derived which were combined synthetically to produce the "one best way" and to predict the times required to perform other operations made up of similar elements.

Two of the basic assumptions made by Taylor in the development of this elemental technique were; that the subdivided motions or elements could be reproduced in a perfect motion pattern by all individuals and that they constituted an additive set.

Development and application.--The Gilbreths further refined the elemental breakdown proposed by Taylor which led to the development of the specialized field of motion study. Job improvement and job standardization programs, presently employed, are two of the major applications of motion study. The elemental breakdowns are used to plan programs of instruction, in designing revised work methods, and for other purposes.

The elemental breakdown has also facilitated a new method of rating. Mundel has developed an "objective rating" system which is only applicable to an element time approach (22, p. 333).

The original standard data technique has given rise to many such systems which Gomberg has divided into two basic categories: "macroscopic," with data formulated in sizable job elements, and "microscopic," with data formulated in minute muscular reactions or therbligs (16, p. 214). The essential difference being that the first refers to groups of motions and the latter to individual motions. Carroll is one of the advocates of the former system in view of its purported consistency and wide adaptability (6, pp. 18-23). Mundel (22, p. 178) and Gillespie (14, p. 77) also tend to favor this method. The competitive "microscopic" systems such as the Methods-Time Measurement (21), Work Factor (28), and those proposed by Holmes (18), Barnes (2), Segur (30), and others have become increasingly popular in recent years. With the exception of Barnes (2, p. 334) each set of data is asserted to have universal application.

Critical views.--The applications of time study techniques and methods based on the validity of the assumptions made concerning element independence have constantly been questioned.

In connection with job improvement and job standardization Gomberg raises the question: "Is there one best method equally applicable to all people?" (16, p. 156). Myers, an English psychologist, expressed the opinion that:

> ...it was supposed that success could be attained by dividing an operation into a number of different parts, in observing and standardizing the movements of a certain operative who performs one of those parts in the quickest time, in observing and standardizing the movements of another operative who performs another of those parts in the quickest time, and so on; finally collecting and stereotyping the different parts thus studied into one whole which is thereupon to be forced upon the worker as "the one best way." (23, p. 81)

This view seems to be supported also by Cohen and Strauss. These psychologists, after conducting their gauze-folding experiment, concluded that there were as many different methods of performing an operation as there were operators (7, p. 151).

Industrial psychologists, especially in recent years, have been giving a great deal of critical consideration to time study. Conrad points out that time studies do not take perception into consideration and further states that elements when in a series such as found in a work cycle are influenced by the behavior pattern (8, pp. 353-358). Ryan (29, pp. 232-235), along with Ghiselli and Brown (13, pp. 268-270), also question the assumption that the time required to perform a given motion or element is not influenced by other motions or elements in the operation.

Niebel and Thuering indicate that extreme caution should be used in the application of motion-time values from standard data systems in setting production standards and that additional studies and statistical verifications are necessary (26, p. 101). While Gomberg hesitantly accepts standard data for bargaining purposes on the basis of its consistency, he establishes four criteria for its scientific validity, two of which are:

2. That the arbitrary divisions into which time study elements are divided are independent of one another.

3. That these elements constitute an additive set (16, pp. 215-216).

Abruzzi comments that standard data and the "one best way" do not take into consideration the correlations between elements or motions and variability in performance (1, p. 218). He further states that time study men assume production rates are statistically stable and constant (1, p. 192). Davidson suggests that element times may not be independent within cycles and that the elemental approach to time study may not be theoretically sound (9, p. 121).

The basic assumption of element independence is further questioned by Jennings (19, p. 16). Wiberg, in studying time study frequency distributions noticed that the same elements differed in the time required under apparently the same conditions (34, p. 216).

The critical views of time study are best summed up by Abruzzi in the following statement:

> It seems to me that we can no longer tolerate taking empirical action about productivity problems on the basis of subjective judgements.... Instead, we need objective principles and procedures so that the estimates we make and the action we take will be sound in a scientific sense (1, p. viii).

Investigations of Element Independence

Barnes and Mundel.--In 1938 Barnes and Mundel found, while studying the time required to position pins in bushings with beveled holes, that certain therbligs in the cycles were interelated hence standard times for certain therbligs cannot be given as independent values (3). In subsequent studies these findings were further verified (4)(5).

Davidson.--Another illustration of interdependency of movements was reported by Davidson based on experiments conducted by graduate students, Moffat and McClure (9, p. 51). The experimenters used a simple task of the "post and washer" variety in reaching the same conclusions as Barnes and Mundel.

Davidson also described some of the experimental work of R. W. McGuire, a research fellow of Ohio State University. McGuire was interested in statistically testing the proposition: "If one of several contemporary standard systems is accurate, then the others cannot be accurate." From data available on the Holmes, Work Factor, and Methods-Time Measurement systems statistical tests revealed that although the Work Factor and the Methods-Time Measurement systems differed less than any other pair, the desparity between them showed that this difference could not be attributed to chance (9, pp. 333-346). Gomberg reports that a later study conducted by L. C. Pigage and L. I. Reis, utilizing a different method, produced the same results (16, p. 228-231). These conclusions seem to refute the assertions of "universal application" of such systems and indirectly jeopardize the assumption of element independence.

Brague, a student of Davidson's, in an analysis of cycle times also analyzed element time variations for part of his data. He found that elements exhibited a greater degree of instability than did cycle times, and concluded that the instability of the element will not produce the same degree of instability in the cycle (9, p. 357). Davidson, in an extension of this study, found, by making a comparison of the total cycle variance with the sum of element variances, that the elements of the operation were dependent (9, p. 358). The data used for the study were taken from industrial time studies. The results were considered inconclusive and further research was recommended. However, on the basis of the findings, Davidson speculated that present methods of establishing time standards on the basis of element independence are erroneous and that confidence intervals for mean cycle times would be narrower than confidence intervals for element time means for a given number of observations (9, p. 358).

Ghiselli and Brown.--A simple key-tapping experiment was conducted by Ghiselli and Brown. It was shown that by the elimination of two of the movements that the cycle time was not reduced by as much as had been expected thereby indicating dependent elements (13, pp. 268-270). They emphasize that an operator works on an operation as a totality, and that each part of the operation affects all other parts.

Abruzzi.--The data collected by time study men in two garment factories were subjected to the Wilks' multivariate test of independence in this investigation. It was found that the elements as originally defined were not independent and that the elements interacted in a complicated network of relationships (1, pp. 143-144). These relationships among the elements were shown to vary with different operators and even the same operator. It was implied on the basis of these findings that a worker tends to organize the work method into a unified whole and does not perform work in terms of individual elements (1, p. 147). This seems to substantiate some of the previous findings of investigations mentioned earlier.

In an effort to establish independent elements the original elements were combined into related groups. It was found that the existence of element independence depends on the number and magnitude of the elements involved, that the amount of grouping required to achieve independence varies with the operator, and that no general rule could be established regarding the size and number of the subdivisions required to establish independence (1, pp. 147-156). Abruzzi suggests that in order to establish element independence for a man-controlled operation that the individual units which are correlated should be grouped. However, he indicates that finding this optimum grouping would not be economical.

Abruzzi further indicated that confidence intervals for element times must be larger than confidence intervals for element groups or cycles which seems to bear out Davidson's speculations which were mentioned earlier (1, p. 193). This would also imply that cycle times should be used for setting production standards. Gillespie reached much the same conclusion on an intuitive basis (14, p. 79).

Davidson questioned Abruzzi's findings on the basis that correlations between elements could have been introduced by the observers making the stop watch studies (11, pp. 11,25). He also questioned the assumption of a normal distribution of the variates (elements) which was made by Abruzzi.

Nadler.--Nadler and Denholm conducted a study with an arrangement of switches to determine what effect the addition or elimination of an element of work would have on established therblig times within a cycle (24, pp. 3-4). It was found that the original total cycle time and surrounding therblig times were significantly affected. It was concluded that the division of an operation into therbligs to obtain therblig times was unwarranted. These findings verified similar studies conducted by Nadler and Wilkes (25, p. 20).

Green.--The primary objective of this investigation was to study the element-time distributions for an industrial operation; one of the secondary objectives was to investigate element independence. The data used in this study was obtained from a research project which has been conducted at the Georgia Institute of Technology under the direction of Doctors Lehrer and Moder. Only one operator working on one manually controlled, short cycle assembly operation was studied.

The characteristics of the element-time distributions were determined by preparing histograms and by calculating the first four moments of these distributions. The element-time distributions were then fitted to the Normal, Log Normal and Pearson Type III curves to determine the nature of the theoretical curve which typified the distributions. The results indicated that the Log Normal curve was generally the curve of the best fit, but that there was a high probability level for the fit of part of the distributions to the Pearson Type III curve (17, p. 57).

A comparison of the sum of the element variances with the total cycle variance revealed evidence of element independence (17, p. 41). Green did not consider these results to be conclusive and recommended that a more rigorous study be made of element independence.

Summary.--Although all these studies have some exploratory value they cannot be considered conclusive because of certain inherent limitations. As far as could be ascertained from the information presented, it was indicated that certain studies were conducted under laboratory conditions, using highly motivated test subjects performing extremely simple tasks. The results which were obtained under these conditions may not be typical of those found in industry. In other studies the basic data were not evaluated statistically and no attempt was made to determine the statistical significance of the results. Limited sample sizes placed further limitations on some of the studies. While Abruzzi's studies appeared to be the most thorough they too are open to question as pointed out by Davidson.

# CHAPTER II

## PURPOSE

Although some of the present-day time study techniques and methodology are based on the assumption of element independence, it has not yet been conclusively determined whether or not the elements of a manually controlled operation are independent. These techniques and methods, which perhaps are not scientifically sound, have proven useful in solving present-day problems however. From the short range viewpoint the requirement of scientific validity does not seem to be absolutely essential, but would be highly desirable. From the long range viewpoint, however, if progress is to be made in the field of work measurement a considerable amount of basic research will be required. This research may produce new concepts that are not only scientifically valid but also provide better solutions to our practical problems. The studies presented in Chapter I indicated that basic research should be conducted concerning the independency of elements.

The primary purpose of this research was to make an exploratory study of the relationships among the elements of a cycle to determine if they were statistically independent. The solution to the problem, therefore, involved accepting or refuting statistically the following null hypothesis: the elements of a cycle are statistically independent.

A secondary purpose was to study the influence of element grouping and the statistical stability of the operator on element independence.

# CHAPTER III

## DATA

Data requirements.--In this exploratory study an attempt was made to avoid as many of the limitations of previous studies as possible. Ideally, it was felt that the study should be based on data which met the following minimum requirements:

1. That the data should be obtained from a micromotion study of a manually controlled industrial operation of the repetitive type.

2. That the data should be obtained for more than one operator on more than one operation. The sample size for each operator should be large enough to obtain a valid statistical result.

3. That the data for each operator studied should be evaluated statistically to determine whether or not the performance data were statistically stable and the level of stability of the operator.

4. That the data should be evaluated statistically to determine the characteristics of the element-time distributions.

These requirements were specified for the following reasons:

1. A micromotion analysis would tend to reduce any errors introduced by observation.

2. A study made on industrial operations performed under factory conditions by representative workers would more closely approximate the actual work situation than laboratory studies using highly motivated test subjects performing extremely simple tasks.

3. The statistical evaluation of the data from the micromotion analysis would indicate the level of statistical stability of each operator which might be useful in establishing element independence. Further, if the characteristics of the element-time distributions were known this would be

useful in testing for element independence since it would not be necessary to make an unfounded assumption concerning these distributions.

4. The actual work situation would be more closely approximated if the number of operators, the number of operations, and the sample sizes were large.

Selection of data.--It was apparent that the amount of work necessary to meet these ideal requirements would make such an investigation prohibitive at the masters level. However, it was found that data were available which approximated the requirements set forth. The data had been collected in various studies in connection with a research project, under the direction of Doctors Lehrer and Moder, in the School of Industrial Engineering at the Georgia Institute of Technology. The objective of the project, begun in 1951, was to analyze statistically the characteristics of an industrial worker's performance pattern on a manually controlled repetitive operation. The operation selected for study was the assembly of a ball point pen.

In conjunction with this project Taft (33) made a micromotion study of nineteen operators on the operation; Summers (31), in the course of his study determined the level of stability of the operators studied by Taft; and Green (17) made a study of the element-time distributions for one of the operators studied by Taft. The data from these studies were selected as the basic data for the present investigation since they approximated the established ideal requirements.

Description of data.--Taft, utilizing a high speed camera (2000 frames per minute), took fifteen thousand feet of film of nineteen operators working on three shifts (33). The operators were engaged in the assembly of a ball point pen. He obtained eight film "shots" of each operator at intervals of approximately one hour. Each "shot" contained from twelve to fifteen complete work cycles.

The operators were not under any financial incentive system, however they were expected to produce between four and five thousand units per day. All operators seem to be highly motivated and personnel relations in the assembly room appeared to be excellent. The general working conditions were clean and well lighted, ventilated and pleasant in appearance.

The work cycle was broken down into the following twelve elements (a detailed description of the operation can be found in the Master of Science Thesis by Taft (35)):

1. Get barrel
2. Place barrel in fixture
3. Get writing unit
4. Place writing unit in barrel
5. Get drive nut
6. Place drive nut on unit
7. Get ferrule
8. Place ferrule over drive nut
9. Get complete unit from fixture
10. Place complete unit in staking device
11. Stake ferrule to secure assembly and remove from the staking device
12. Aside assembled unit to container.

The film of each of the nineteen operators was analyzed frame by frame. Each element was broken down into therbligs and the element end points were determined visually, then recorded in terms of frame number. A sample film analysis sheet is shown in Figure 1. These analysis sheets provided the basic data for the present investigation.

Summers was interested in comparing the statistical stability of an operator's cycle time with the characteristics of his work-time distribution (31). In order to do this it was necessary to establish some criterion for the level of stability of that operator's observed times. The total variance between an operator's period ("shot") averages has two components; one due to the variance within a period of observation, and the other due to the shift in the operator's level of performance from period to period. An analysis of variance was performed to break the variance of cycle times into its two components. The "Variance between Periods" component was used as a measure of the level of stability for each operator. The results of these calculations, made on data from Taft's study, were used in the present investigation in the selection of operators for study. (33, pp. 24-27).

Green's study, described in Chapter I, was also based on the data from the micromotion analysis of the operation made by Taft (17). Since the present investigation is based on the same data it was felt that his results, even though only one operator was studied, would provide a sound base on which an assumption could be made concerning the theoretical distributions of the variates (elements). In the application of Wilks' multivariate test of independence, which was used in the present investigation, certain assumptions have to be made concerning the distributions of the variates (elements).

Analysis Sheet for T-600 Ball Point Pen

Operator Carrie Time 12:05A Cycle L Film No. 50

Analyst WL Date of Analysis 6 May 53

Time Unit K

| | Frame No. LH | Subtracted Time LH | RH | Frame No. RH | Remarks |
|---|---|---|---|---|---|
| | 962 | | | 962 | |
| Get Bbl-TE, ST&G | 937 | 25 | 14 | 948 | |
| Place Bbl-TL, P, A&RL | 905 | 32 | 39 | 909 | |
| Get Unit-TE, St&G | 883 | 22 | 19 | 890 | |
| Place Unit-TL, P, A&RL | 835 | 48 | 51 | 839 | |
| Get Dr. Nut-TE, ST,&G | 813 | 22 | 18 | 821 | |
| Place Dr. Nut-TL, P, A&RL | 748 | 65 | 74 | 757 | |
| Get Ferrule-TE, ST&G | 732 | 16 | 17 | 740 | |
| Place Ferrule-TL, P, A&RL | 693 | 39 | 20 | 720 | |
| Get Comp. Unit-TE&G | 684 | 9 | 8 | 712 | |
| Place Comp. Unit-DA, TL&P | 645 | 39 | 30 | 682 | |
| Stake-A, H&DA | 614 | 31 | 68 | 614 | |
| Aside-TL&RL | | | | | Transferred pens to LH |

Figure 1. Sample Film Analysis Sheet

Summary.--Since this investigation was based on data collected previously nothing could be done to control the factors involved in the experimental situation. Therefore, the control accomplished previously will also be an inherent part of this investigation. However, by using these data it was possible to make this exploratory investigation which otherwise would have been prohibitive due to time limitations.

# CHAPTER IV

# PROCEDURE

The general procedure followed in making this investigation was to select two operators for study, assemble the data for these operators for statistical evaluation, and to statistically evaluate these assembled data.

## Selection of Operators

The two operators selected for study in this investigation were chosen from the nineteen which were included in the micromotion study made by Taft (33).

Criteria for selection.--The selection of the two operators was based on two criteria; first, the level of statistical stability exhibited by the operator, and second, the amount of data available for the operator in the form of complete work cycles from the micromotion analysis. It was necessary to have some indication of the level of statistical stability of the operators since one of the secondary objectives of the investigation was to attempt to assess the effect of this factor on element independence. Further, it seemed that the strongest indication of the resultant effect of this factor on element independence could be obtained when the two operators selected represented as nearly as possible the extreme conditions of stability exhibited by the group. It was also recognized that equal consideration should be given to the amount of data available for the two operators which were to be selected since the most valid statistical results could be obtained when the sample sizes were as large as possible. It was also felt that the selection of operators in accordance with these criteria would tend to randomize the effects of other factors, such as the human characteristics of the operators and the thoroughness of the film analysis, which might affect the selection of the data.

The selection of the operators in accordance with these two criteria was based on information obtained from the studies of Summers (31) and Taft (33). An indication of the level of stability of each of the operators was established in Summers' study by calculating the "variance between periods (shots)." The determined magnitude of this component was then used as the measure of the level of stability of the operator. Although this study was based on a modified cycle where the first and last elements had been omitted due to variations in methods between operators, it was felt that the results obtained were indicative of the level of stability of the operators. The first step in the selection process was to determine which of the operators represented the extreme conditions of stability. This was done by examining the results obtained by Summers (31, pp. 24-27). The second step was to determine the amount of data that was available for each of these operators in the form of complete work cycles. This was done by making an examination of the film analysis sheets prepared for these operators by Taft. The final selection of the two operators was then made in accordance with the established criteria.

The operators.--The two operators selected for study were those designated by Summers as Q and K (31, p. 15). Operator Q represented the stable operator and operator K the unstable operator. The following results were obtained by Summers in his analysis of variance on the data for these operators (31, pp. 26-27):

| Operator | Variance between Periods | Variance within Periods |
|---|---|---|
| Q | $\sigma m^2 = 28.31$ | $\sigma w^2$ 392 |
| K | $\sigma m^2 = 444.65$ | $\sigma w^2$ 653 |

Both operators had been employed on the job for a period of ten months when the film study was made. Company records revealed that a worker could reach the level of maximum production in two months. Operator Q was employed on the third shift and operator K on the second shift. The operators on the third shift had relatively little supervision as compared to the two other shifts.

## Assembly of Data

The film analysis sheets (Figure 1.) which were prepared by Taft (33) provided the basic data for operators Q and K. These sheets had been assembled by "shots" for each operator and were on file in the school of Industrial Engineering at the Georgia Institute of Technology along with the film for each "shot." Each of these eight "shots" contained from twelve to fifteen complete cycles.

Elimination of variables.--Each of the 12 elements in the original breakdown of the operation had been broken down into therbligs which were recorded on the film analysis sheets. A study was made of these sheets for each operator in order to determine what variations occurred in each cycle. This information was then used as a guide in making a study of the film "shots" of each operator. It was found that some of the cycles contained variables which could not be considered as a normal part of the work cycle. It seemed desirable to eliminate the work cycles containing any of these variables since they appeared to be due to assignable causes.

The variables which were selected and eliminated are listed below along with the principal reasons for rejection:

1. Inspection delay - a prolonged visual or physical inspection of an assembled or a sub-assembled part. The workplace and required motions were methodized to such an extent and the parts were of such a uniform nature that assembly normally proceeded with little if any inspection at all.

2. Bad part - occurred when an assembly operation could not be accomplished with the part originally selected necessitating the replacement of the part with another. This represented a major departure from the normal work cycle which to some extent could be corrected. Theoretically this could be eliminated by better quality control.

3. Part stuck in staker - an occurrence which was due to the improper functioning of the mechanical staking device. This source of variation was readily apparent and subject to elimination.

5. Distraction - occurred when the operator's attention was purposely and noticeably directed to an object other than the assembly operation; talking to another person and reading while engaged in the assembly operation.

The variables selected and eliminated were easily recognizable and represented major departures from the normal work cycle. No attempt was made to eliminate those cycles which contained minor departures, such as momentary fumbles, slight delays in positioning parts, and dropping extra parts, since they were considered to be an inherent part of the operation due to the size of the parts involved. This elimination procedure was adopted so that the data would reflect the actual work situation as closely as possible. It was found, after cycles containing variables had been eliminated, that there were seventy-two complete work cycles available for study for operator Q and fifty-nine for operator K.

Element grouping.--The original twelve elements of the operation were grouped as shown in Figure 2 to form a five and a two element breakdown of the operation. These breakdowns were established in order that the possible effects of grouping on element independence could be studied, and so that the number of elements would be reduced to a number which would facilitate the application of Wilks' multivariate test of independence.

The five and the two element breakdowns of the operation were chosen after making a careful study of the operation and after considering other possible combinations of the original twelve elements. These particular groupings were chosen because they seemed to represent the most definite and natural "breaks" in the operation, the elements in the established groups seemed to be related, if elements of this size showed indications of not being independent it would seem unnecessary to break the operation into smaller elements, and they represented breakdowns similar to those used in industry. It was recognized however that the validity of the grouping process was dependent on the validity of the original twelve element breakdown, which might not represent the "true" subdivisions of the operation. This was not considered to be a critical factor in the present situation by virtue of the fact that the original breakdown was made on the basis of a detailed film analysis (33).

| Original Twelve Element Breakdown | Five Element Breakdown | Two Element Breakdown |
|---|---|---|
| 1. Get barrel | 1. Get barrel, place barrel in fixture | 1. Get barrel, place barrel in fixture, get writing unit, place writing unit in barrel, get drive nut, place drive nut on unit, get ferrule, place ferrule over drive nut |
| 2. Place barrel in fixture | 2. Get writing unit, place writing unit in barrel | 2. Get complete unit from fixture, place unit in staking device, stake ferrule to secure assembly and remove from staker, aside assembled unit to container |
| 3. Get writing unit | 3. Get drive nut, place drive nut on unit | |
| 4. Place writing unit in barrel | 4. Get ferrule, place ferrule over drive nut | |
| 5. Get drive nut | 5. Get complete unit from fixture, place unit in staking device, stake ferrule to secure assembly and remove from the staker, aside assembled unit to container | |
| 6. Place drive nut on unit | | |
| 7. Get ferrule | | |
| 8. Place ferrule over drive nut | | |
| 9. Get complete unit from fixture | | |
| 10. Place unit in staking device | | |
| 11. Stake ferrule and remove assembled unit from staker | | |
| 12. Aside assembled unit to container | | |

Figure 2. Element Grouping Procedure

<u>Element times</u>.--The element time in this investigation was considered to be the composite time required for both hands to perform a sub-task into which the operation had been divided. It was obtained by taking reading for the hand which was last to complete the preceding element and subtracting the reading for the hand which was the last to complete the element being considered. The element times which provided the basic data for the present study were recorded in terms of frame numbers and were taken directly from the film analysis sheets for operators Q and K. The element times were recorded for each operator for each breakdown by "shots."

Evaluation of Data

<u>The test</u>.--The data assembled for this investigation was subjected to Wilks' multivariate test of independence (36, pp. 242-245). This test was intended to determine whether the members of a set of variables (elements) which are normally distributed, are independent of one another. The alternative conclusion to independence was that the variables (elements) were correlated. The test statistic was computed from

$$L = \frac{\left|S^1\right|^{n/2}}{\left|S^2\right|^{n/2}}$$

where L represents the likelihood ratio, n the sample size, $S^1$ denotes the covariance matrix in which the presence of correlation is assumed, and $S^2$ the covariance matrix where the absence of correlation is assumed. The L value was computed after the entries for the covariance matrices had been determined and the matrices evaluated numerically (that is, their determinant values obtained). The computed L value represented the likelihood that the hypothesis of independence was true; values of L close to one indicate that it was true, while values of L close to zero indicate that it was false.

It was more convenient, however, to compute the L* value ($L^* = -2\text{Log}_e L$), since $-2\text{Log}_e L$ has approximately the same distribution as chi-square with large samples (36). Therefore, the L* value could be used to enter the chi-square tables for $p(p-1)/2$ degrees of freedom, where p represents the number of elements, to determine the significance of the observed result. The probability of obtaining the value of L* when the hypothesis of independence was true could then be read directly from the tables.

Test application.--A likelihood ratio was developed for each of the eight "shots" of each operator for both the five and the two element breakdowns. In addition a likelihood ratio was developed for both breakdowns by combining the data from each of the eight "shots" of each operator. It seemed desirable to establish a likelihood ratio for each "shot" in order that a test of element independence could be made for each of these periods. This would tend to reduce any masking effects that might be introduced by combining the data from all eight "shots."

The element-time distributions of operators Q and K were transformed so that they approximated normal distributions. This was accomplished by taking the common logarithms of each element reading for each breakdown. It was necessary to make this transformation because

Wilks' test was intended for application in cases where the variables approximate a normal distribution. The assumption, that the element-time distributions of operators Q and K approximated Log Normal distributions, was based on the finding of Green (17). This assumption seemed to be particularly well founded since Green based his study on the same basic data for operator Q as was used in this study.

The likelihood ratios were developed from a series of matrices. First, it was necessary to develop cross product matrices from which corresponding entries in the covariance matrices could be determined. The cross product matrices were developed as shown in Figure 3. All these matrices and those developed from them were symmetrical. Therefore, it was only necessary to compute the upper triangular halves of the numerator matrices in order to evaluate them since each $X_iX_j$ value was equal to each $X_jX_i$ value. The denominator matrices had entries only on the main diagonals since the covariance terms were zero when the absence of correlation was assumed. Further, since the corresponding entries of the main diagonals of the numerator and denominator matrices were equal it was only necessary to record the numerator matrices in order to obtain the L and L* values. The sample calculations involved in the development of the cross product matrices are shown as Figure 4 in the Appendix.

The covariance matrices were developed from each of the cross product matrices. The corresponding entries in the covariance matrices were computed from

$$
\begin{array}{cccccc}
 & X_1 & X_2 & X_3 & \cdots\cdots & X_p \\
X_1 & \sum_{c=1}^{n} X_{1c}X_{jc} & \sum_{c=1}^{n} X_{1c}X_{2c} & \sum_{c=1}^{n} X_{1c}X_{3c} & \cdots\cdots\cdots & \sum_{c=1}^{n} X_{1c}X_{pc} \\
X_2 & & \sum_{c=1}^{n} X_{2c}X_{2c} & \sum_{c=1}^{n} X_{2c}X_{3c} & \cdots\cdots\cdots & \sum_{c=1}^{n} X_{2c}X_{pc} \\
X_3 & & & \sum_{c=1}^{n} X_{3c}X_{3c} & \cdots\cdots\cdots & \sum_{c=1}^{n} X_{3c}X_{pc} \\
\vdots & & & & \ddots & \vdots \\
X_p & & & & & \sum_{c=1}^{n} X_{pc}X_{pc}
\end{array}
$$

$p$ = number of elements (variables)

$n$ = sample size

$x$ = reading for the pth element in the cth cycle

Figure 3. Cross Product Matrix

$$X_iX_j = \frac{n\sum_{c=1}^{n} X_{ic}X_{jc} - \sum_{c=1}^{n} X_{ic}\sum X_{jc}}{n}$$

where $X_i$ and $X_j$ referred to readings on elements i and j, respectively, in the cth cycle. The sample calculations involved in the development of the covariance matrices are shown as Figure 5 in the Appendix.

It was necessary to evaluate each of the covariance matrices numerically in order to determine the L and L* values. This involved obtaining the determinant values of each of the covariance matrices in the numerator and denominator of each likelihood ratio that was developed (12). It was necessary to reduce the numerator matrices as shown in Figure 6 in order to obtain their determinant values. The determinant values of the numerator matrices were then obtained by squaring the product of the entries on the main diagonals of these reduced matrices. The determinant values of the denominator matrices were obtained directly by computing the product of the entries on the main diagonals of the covariance matrices. The L and L* values were then calculated using the determinant values which were obtained for the covariance matrices. The sample calculations involved in computing the L and L* values are shown as Figure 7 in the Appendix.

The calculated L* values were then compared to the chi-square tables to determine the significance of the observed results.

All the basic data used in this investigation and all the accompanying calculations made in the application to these data are on file in the School of Industrial Engineering at the Georgia Institute of Technology. Only a limited amount of these data were included in this presentation due to its voluminous form.

$$X_1^*X_1 = \sqrt{X_1X_1}\ ,\quad X_1^*X_2 = \frac{X_1X_2}{X_1^*X_1}\ ,\ \dots\dots\dots\dots\dots\dots\ X_1^*X_p = \frac{X_1X_p}{X_1^*X_1}$$

$$X_2^*X_2 = \sqrt{X_2X_2 - (X_1^*X_2)^2}\ ,\quad X_2^*X_3 = \frac{X_2X_3 - (X_1^*X_2)(X_1^*X_3)}{X_2^*X_2}\ ,\ \dots\ X_2^*X_p = \frac{X_2^*X_p - (X_1^*X_2)(X_1^*X_p)}{X_2^*X_2}$$

$$X_3^*X_3 = \sqrt{X_3X_3 - (X_1^*X_3)^2 - (X_2^*X_3)^2},\ \dots\dots\dots\dots\ X_3^*X_p = \frac{X_3X_p - (X_1^*X_3)(X_1^*X_p) - (X_2^*X_3)(X_2^*X_p)}{X_3^*X_3}$$

$$\vdots$$

$$X_p^*X_p = \sqrt{X_pX_p - (X_1X_p)^2 - (X_2X_p)^2 - \dots - (X_{p-1}X_p)^2}$$

$X_iX_j$ = Corresponding Entries in Covariance Matrices in the Numerator of the Likelihood Ratios

$X_i^*X_j$ = Entries in the Reduced Covariance Matrices

$p$ = Number of Elements

Figure 6. Procedure Followed in Reducing Entries of Covariance Matrices for Evaluation

# CHAPTER V

## DISCUSSION OF RESULTS

The calculated L* values which were obtained for each breakdown for each operator by shots were used to test for element independence at the five per cent significance level. The L* values obtained from the application of Wilks' test and the corresponding probability values from the chi-square tables are shown for operators Q and K in Table 1 and 2, respectively. The underlined probability values indicate those cases in which there was found to be evidence of correlation.

Element independence.--The results obtained by shots indicated that there was evidence of correlation for both the five and the two element breakdowns. However, it was found when the data for all eight shots for an operator were combined a masking effect was introduced and there was no evidence of correlation.

There were several possible reasons why the presence of correlation was found to exist; first, it could have been caused by the operator re-organizing the work from period to period, second, the original twelve element breakdown and the subsequent groupings may not have represented "true" subdivisions of the operation, especially when the operator tended to re-organize the work from period to period, third, these shots may have contained variables which tended to produce the resultant correlations among the elements, and fourth, it may have been a combination of these factors.

Table 1. Summary of Results from Wilks' Test of Independence
Operator Q

| Shot No. | Time | No. Cycles $n$ | $L^*$ Value Breakdowns Five | $L^*$ Value Breakdowns Two | Prob. of Obtaining $L^*$ Value When Elements Are Independent. (Chi-Square - %) Five | Prob. of Obtaining $L^*$ Value When Elements Are Independent. (Chi-Square - %) Two |
|---|---|---|---|---|---|---|
| Start | 11:40A | | | | (D.F. = 10) | (D.F. = 1) |
| 1 | 12:20A | 10 | 27.458 | 0.702 | $0.1 < L^* < 0.5$ | $30 < L^* < 50$ |
| 2 | 12:55A | 8 | 9.699 | 0.030 | $30 < L^* < 50$ | $80 < L^* < 90$ |
| 3 | 2:20A | 10 | 28.831 | 3.149 | $0.1 < L^* < 0.5$ | $5 < L^* < 10$ |
| 4 | 2:50A | 3 | 18.534 | 1.781 | $2.5 < L^* < 5.0$ | $10 < L^* < 20$ |
| Rest | 3:20A | | | | | |
| 5 | 3:50A | 10 | 34.350 | 0.607 | $0.0 < L^* < 0.1$ | $30 < L^* < 50$ |
| Rest | 5:00A | | | | | |
| 6 | 5:25A | 11 | 4.942 | 0.111 | $80 < L^* < 90$ | $70 < L^* < 75$ |
| 7 | 5:55A | 10 | 14.168 | 0.184 | $10 < L^* < 20$ | $50 < L^* < 70$ |
| 8 | 6:27A | 10 | 10.032 | 2.206 | $30 < L^* < 50$ | $10 < L^* < 20$ |
| Stop | 7:00A | | | | | |
| All Shots | -- | 72 | 7.897 | 0.5134 | $50 < L^* < 70$ | $30 < L^* < 50$ |

Table 2. Summary of Results from Wilks' Test of Independence
Operator K

| Shot No. | Time | No. Cycles n | L* Value Breakdowns Five | L* Value Breakdowns Two | Prob. of Obtaining L* Value When Elements Are Independent. (Chi-Square - %) Five | Prob. of Obtaining L* Value When Elements Are Independent. (Chi-Square - %) Two |
|---|---|---|---|---|---|---|
| Start | 3:20P | | | | (D.F. = 10) | (D.F. = 1) |
| 1 | 3:40P | 8 | 11.948 | 1.197 | 25 < L* < 30 | 25 < L* < 30 |
| 2 | 4:30P | 8 | 14.955 | 2.986 | 10 < L* < 20 | 5 < L* < 10 |
| 3 | 5:20P | 11 | 8.150 | 0.384 | 50 < L* < 70 | 50 < L* < 70 |
| Rest | 6:10P | | | | | |
| 4 | 7:00P | 5 | 42.794 | 5.378 | 0.0 < L* < 0.1 | 2 < L* < 2.5 |
| 5 | 7:40P | 6 | 31.075 | 0.329 | 0.0 < L* < 0.1 | 50 < L* < 70 |
| 6 | 8:35P | 8 | 8.193 | 0.00166 | 50 < L* < 70 | 95 < L* < 97.5 |
| Rest | 9:05P | | | | | |
| 7 | 9:20P | 6 | 35.447 | 4.365 | 0.0 < L* < 0.1 | 2.5 < L* < 5 |
| 8 | 10:45P | 7 | 23.101 | 1.895 | 1.0 < L* < 2.0 | 10 < L* < 20 |
| Stop | 11:40P | | | | | |
| All Shots | -- | 59 | 6.630 | 0.213 | 75 < L* < 80 | 50 < L* < 70 |

The variance of element independence between shots suggest that the nature and the extent of the relationship among the elements did not remain constant from period to period for the same operator. This was evidenced when a comparison was made of the corresponding covariance entries (those off the main diagonal) in the covariance matrices in which the presence of correlation was assumed. The corresponding entries were found to be quite different in sign and also magnitude for the same operator, even for shots containing the same number of cycles. These findings were also reflected in the different determinant values for these matrices. The determinant values and the corresponding L values obtained for operators Q and K are shown in Table 3 and 4, respectively. These differences in the relationship among the elements were also found to exist between operators when compared on the basis of corresponding periods in the shift. This implied that an operator tended to re-organize the work from period to period and that the nature and extent of the relationship among the elements depended on the operator. This was also reflected by the different probability values when a shot to shot comparison was made for each operator and also between operators. It was particularly interesting to note the pattern which was followed by operator K. With the exception of the initial shot, evidence of correlation was found in each shot which followed a rest period. This organization pattern was further emphasized when an examination of the project log revealed that there was a suspension of operations from 9:20 p.m. until 10:35 p.m. due to a lack of parts. This period occurred just prior to the time that film shot number eight was made of Operator K. This shot was also found to exhibit evidence of correlation.

Table 3. Determinant Values and L Values for Operator Q

| Shot No. | No. Cycles | Numerator | Denominator | L Value |
|---|---|---|---|---|
| | | (Five Element Breakdown) | | |
| 1 | 10 | $211,679 \times 10^{17}$ | $3,306,470 \times 10^{17}$ | $(0.0604)^{10/2}$ |
| 2 | 8 | $343,216 \times 10^{15}$ | $1,155,280 \times 10^{15}$ | $(0.2791)^{8/2}$ |
| 3 | 10 | $838,583 \times 10^{16}$ | $1,503,327 \times 10^{17}$ | $(0.05578)^{10/2}$ |
| 4 | 3 | $344,451 \times 10^{2}$ | $1,671,460 \times 10^{5}$ | $(0.00206)^{3/2}$ |
| 5 | 10 | $134,917 \times 10^{16}$ | $4,202,890 \times 10^{16}$ | $(0.03210)^{10/2}$ |
| 6 | 11 | $109,685 \times 10^{18}$ | $171,968 \times 10^{22}$ | $(0.6378)^{11/2}$ |
| 7 | 10 | $108,997 \times 10^{16}$ | $450,162 \times 10^{16}$ | $(0.2421)^{10/2}$ |
| 8 | 10 | $532,208 \times 10^{16}$ | $1,453,072 \times 10^{16}$ | $(0.3663)^{10/2}$ |
| All | 72 | $182,737 \times 10^{22}$ | $203,738 \times 10^{22}$ | $(0.8960)^{72/2}$ |
| | | (Two Element Breakdown) | | |
| 1 | 10 | $340,343 \times 10^{2}$ | $368,952 \times 10^{2}$ | $(0.9225)^{10/2}$ |
| 2 | 8 | $456,435 \times 10^{2}$ | $457,788 \times 10^{2}$ | $(0.9970)^{8/2}$ |
| 3 | 10 | $490,312 \times 10^{3}$ | $670,617 \times 10^{3}$ | $(0.7311)^{10/2}$ |
| 4 | 3 | 44,521 | 80,640 | $(0.5521)^{3/2}$ |
| 5 | 10 | $774,699 \times 10^{4}$ | $823,195 \times 10^{4}$ | $(0.9411)^{10/2}$ |
| 6 | 11 | $941,189 \times 10^{3}$ | $950,729 \times 10^{3}$ | $(0.9899)^{11/2}$ |
| 7 | 10 | $212,684 \times 10^{3}$ | $213,093 \times 10^{3}$ | $(0.0184)^{10/2}$ |
| 8 | 10 | $122,622 \times 10^{3}$ | $153,246 \times 10^{3}$ | $(0.8002)^{10/2}$ |
| All | 72 | $919,873 \times 10^{5}$ | $926,452 \times 10^{5}$ | $(0.9929)^{72/2}$ |

Table 4. Determinant Values and L Values for Operator K

| Shot No. | No. Cycles | Determinant Values Numerator | Denominator | L Value |
|---|---|---|---|---|
| | | (Five Element Breakdown) | | |
| 1 | 8 | $459,124 \times 10^{18}$ | $2,048,250 \times 10^{18}$ | $(0.2242)^{8/2}$ |
| 2 | 8 | $128,347 \times 10^{18}$ | $833,802 \times 10^{18}$ | $(0.1539)^{8/2}$ |
| 3 | 11 | $560,146 \times 10^{17}$ | $1,176,010 \times 10^{17}$ | $(0.4763)^{11/2}$ |
| 4 | 5 | $369,362 \times 10^{9}$ | $1,947,620 \times 10^{14}$ | $(0.0000019)^{5/2}$ |
| 5 | 6 | $747,213 \times 10^{15}$ | $1,332,520 \times 10^{17}$ | $(0.0056)^{6/2}$ |
| 6 | 8 | $275,029 \times 10^{17}$ | $766,656 \times 10^{17}$ | $(0.3587)^{8/2}$ |
| 7 | 6 | $823,633 \times 10^{15}$ | $3,008,010 \times 10^{15}$ | $(0.0027)^{6/2}$ |
| 8 | 7 | $618,718 \times 10^{16}$ | $1,683,890 \times 10^{17}$ | $(0.03674)^{7/2}$ |
| All | 59 | $251,086 \times 10^{23}$ | $280,996 \times 10^{23}$ | $(0.8936)^{7/2}$ |
| | | (Two Element Breakdown) | | |
| 1 | 8 | $363,514 \times 10^{4}$ | $422,250 \times 10^{4}$ | $(0.8609)^{8/2}$ |
| 2 | 8 | $537,822 \times 10^{4}$ | $781,519 \times 10^{4}$ | $(0.6882)^{8/2}$ |
| 3 | 11 | $280,140 \times 10^{3}$ | $290,081 \times 10^{3}$ | $(0.9657)^{11/2}$ |
| 4 | 5 | $223,310 \times 10^{2}$ | $655,526 \times 10^{2}$ | $(0.3407)^{5/2}$ |
| 5 | 6 | $152,350 \times 10^{3}$ | $160,938 \times 10^{3}$ | $(0.9466)^{6/2}$ |
| 6 | 8 | $323,762 \times 10^{4}$ | $323,799 \times 10^{4}$ | $(0.9998)^{8/2}$ |
| 7 | 6 | $589,759 \times 10^{2}$ | $1,221,820 \times 10^{2}$ | $(0.4827)^{6/2}$ |
| 8 | 7 | $304,213 \times 10^{3}$ | $398,896 \times 10^{3}$ | $(0.7626)^{7/2}$ |
| All | 59 | $778,499 \times 10^{5}$ | $781,325 \times 10^{5}$ | $(0.9964)^{59/2}$ |

The change in relationship among the elements from period to period may have further complicated the matter of element independence since the "true" definitions of the elements would change from period to period for the same operator. Therefore, the definitions established for the elements in this investigation may not have been representative of the "true" subdivisions of the operation in every case; one part of a "true" subdivision of the operation may have been included in one element group and the other part in another. This same situation may have been encountered when the film analysis was made. Resulting errors may have been introduced since the original twelve elements, as defined, may have been artificial and hard to distinguish.

Only a limited number of variables were eliminated from the data in this investigation although it was found that many of the work cycles contained minor variations. These variations were introduc3d by the operator in the form of momentary fumbles, slight delays, and extraneous movements. They were found to occur with such frequency and appeared to be so closely associated with the size and nature of the parts involved in the operation that they were considered to be an inherent part of the operation. The work cycles containing these variables were not eliminated since it was desired to preserve the actual work situation as closely as possible. A re-examination was made of the film analysis sheets for those shots in which there was found to be evidence of correlation. The film analysis sheet for those shots evidencing correlation were then compared with those in which there was found to be no evidence of correlation. Although variables were found to occur in all shots, they appeared to be concentrations of variables in those shots in which evidence of correlation was found. These concentrations of variables did not appear to follow any particular pattern, they were of different types, different in number and duration, and were introduced differently in the cycle. These variables were not visually apparent when the film for each shot was reviewed, however, the film was not analyzed on a frame to frame basis.

<u>The effect of grouping on element independence</u>.--It was found with the exception of two cases for operator K, that the grouping procedure produced elements which did not exhibit evidence of correlation. This result suggested that there was a better chance of achieving independence when the elements were large and few in number. This was evidenced when a review was made of the film analysis sheets; minor departures from the work method seemed to be absorbed in the larger elements, while the smaller subdivisions did not mask these departures. This was also evidenced statistically by the entries in the covariance matrices in the numerators of the likelihood ratios for the five and two element breakdowns. The entries off the main diagonal (covariance terms) in the five element breakdown reflected the correlations between the small subdivisions. These correlations were made a part of the variation within the larger subdivisions developed in the two element grouping process by being absorbed into the main diagonal terms (the variance terms). The net effect was to reduce the difference between the determinate values of the numerator and denominator matrices thus yielding in most cases a higher probability value.

It was noted that in some cases, where there was no evidence of correlation for the shot, that the probability value decreased as a result of the grouping procedure. The reason for this was that the number of degrees of freedom was reduced at a faster rate than the L* value; the net result being a smaller probability value. This indicated that there was an optimum grouping plan beyond which a further reduction in the number of element groups could not be expected to improve the degree of independence.

It was also noted that although the elements in the shots did not exhibit evidence of correlation they were shown to have different degrees of correlation. This was evidenced by the differences in the determinant and probability values.

The results also indicated that the grouping procedure would not necessarily produce independent elements in every case even for the same operator. This situation was exhibited by the data for operator K.

Effect of operator stability on element independence.--It was found on the five element breakdown that there was evidence of correlation for both the stable and unstable operator. It was found on the two element breakdown that only the data for operator K, the unstable operator, revealed evidence of correlation. However, the grouping process did produce elements which did not exhibit evidence of correlation for two of the shots for operator K, and the degree of correlation was decreased in the other two.

Both operators appeared to exhibit the same characteristics; re-organization of the work from period to period, evidence of correlation for those shots which were found to include concentrations of variables, improved probability values from the grouping process for those shots previously evidencing correlation, and similar determinant and probability values indicating that the degree of correlation in each case was similar. Therefore, on the basis of the observed results operator stability did not appear to have any appreciable effect on element independence. This was further evidenced when the data from all eight shots were combined for each of the operators; it was found that there was no evidence of correlation in either case.

# CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

The purpose of this investigation was to study the relationship among the elements of a cycle to determine if they were statistically independent. An attempt was made to approximate as closely as possible the actual work situation as found in industry. This seemed desirable since this represented the area where time study techniques and methods based on the assumption of element independence, have their greatest application.

The null hypothesis tested in this investigation was: the elements of a cycle are statistically independent. The preceding results indicate that the hypothesis must be rejected since it was found that the elements as defined in both the five and the two element breakdowns exhibited evidence of correlation.

The results and conclusions from this investigation must be viewed in the light of several inherent limitations. The investigation covered only one operation in one plant, thus restricting the general conclusions which could be drawn concerning different operations in different locations. Only two operators were studied in this investigation and these operators were working on different shifts. The data used in this investigation were collected in previous studies, therefore, nothing could be done to control the factors involved in the experimental situation. Further, the information concerning the factors involved in the experimental situation was limited. This information might have been helpful in the interpretation of the results. There was a limited amount of data available for study when considered on the shot basis. The variables which were eliminated from the data were those which represented major departures from the usual work method.

Specific conclusions.--The following specific conclusions were made as a result of the investigation:

1. There was an indication that the degree of correlation among the elements of a cycle does not remain constant for the same operator during the work shift.

2. The nature and extent of correlation among the elements of a work cycle from period to period appeared to depend on the operator.

3. It appeared that in those shots where the degree of correlation among the elements was found to be the highest, there was a concentration of variables.

4. There was an indication that the degree of correlation among the elements of the five element breakdown was decreased by combining these elements into a two element breakdown of the operation.

5. The grouping process did not decrease the degree of correlation among the elements to the same extent for the same operator on data from different shots.

6. The stable and unstable operators exhibited similar characteristics in regard to the degree and extent of correlation among the elements of the work cycle.

Recommendations.--The results of this investigation indicated that further exploratory studies should be made to determine the coefficients of correlation among the elements of the work cycle from shot to shot. The data for the present investigation could be used in such a future study. In conjunction with this study the film should be re-analyzed and all variables classified by elements by shots. This information could then be used to determine how the variables affected the correlations between the various elements from period to period. Additional information concerning the work patterns of the operators could also be gained from such a study. Further, it is recommended that the data for the original twelve element breakdown of the operation be treated in a similar manner and compared with the five and two element breakdown.

Any future studies of element independence should be based on film which is made during the entire work shift over a period of days. Such a film would provide a greater amount of data for an operator, giving a clearer picture of the change in relationship among the elements during the work shift and also from day to day. If such a film were made of a number of operators on the same shift a comparison could be made of the work patterns for different operators.

# A P P E N D I X

Operator K, Shot Number 4
Five Element Breakdown

| Element Number | Element Readings by Cycles (n=5) 1 | 2 | 3 | 4 | 5 | Sum of Element Readings |
|---|---|---|---|---|---|---|
| 1 | 279 | 290 | 279 | 267 | 290 | 1405 |
| 2 | 639 | 607 | 512 | 663 | 342 | 2763 |
| 3 | 322 | 597 | 389 | 538 | 591 | 2437 |
| 4 | 342 | 312 | 398 | 342 | 431 | 1825 |
| 5 | 597 | 455 | 550 | 498 | 574 | 2674 |

Calculations of Entries

$$X_1X_1 = \sum_{c=1}^{n} X_{1c}X_{1c} = (279)^2 + (290)^2 + (279)^2 + (267)^2 + (290)^2 = 395{,}171$$

$$X_1X_2 = \sum_{c=1}^{n} X_{1c}X_{2c} = (279)(639) + (290)(607) + (279)(512) + (267)(663) + (290)(342) = 773{,}360$$

$$X_1X_3 = \sum_{c=1}^{n} X_{1c}X_{3c} = (279)(322) + (290)(597) + (279)(389) + (267)(538) + (290)(591) = 686{,}535$$

$$X_1X_4 = \sum_{c=1}^{n} X_{1c}X_{4c} = (279)(342) + (290)(312) + (279)(398) + (267)(342) + (290)(431) = 513{,}244$$

$$X_1X_5 = \sum_{c=1}^{n} X_{1c}X_{5c} = (279)(597) + (290)(455) + (279)(550) + (267)(498) + (290)(574) = 751{,}389$$

Figure 4. Sample Calculations Involved in the Development of the Cross Product Matrices

Operator K, Shot Number 4
Five Element Breakdown

$$X_2X_2 = \sum_{c=1}^{n} X_{2c}X_{2c} = (639)^2 + (607)^2 + (512)^2 + (663)^2 + (342)^2 = 1{,}595{,}447$$

$$X_2X_3 = \sum_{c=1}^{n} X_{2c}X_{3c} = (639)(322) + (607)(597) + (512)(389) + (663)(538) + (342)(591) = 1{,}326{,}121$$

$$X_2X_4 = \sum_{c=1}^{n} X_{2c}X_{4c} = (639)(342) + (607)(312) + (512)(398) + (663)(342) + (342)(431) = 985{,}846$$

$$X_2X_5 = \sum_{c=1}^{n} X_{2c}X_{5c} = (639)(597) + (607)(455) + (512)(550) + (663)(498) + (342)(574) = 1{,}465{,}750$$

$$X_3X_3 = \sum_{c=1}^{n} X_{3c}X_{3c} = (322)^2 + (597)^2 + (389)^2 + (538)^2 + (591)^2 = 1{,}250{,}139$$

$$X_3X_4 = \sum_{c=1}^{n} X_{3c}X_{4c} = (322)(342) + (597)(312) + (389)(398) + (538)(342) + (591)(431) = 889{,}927$$

$$X_3X_5 = \sum_{c=1}^{n} X_{3c}X_{5c} = (322)(597) + (597)(455) + (389)(550) + (538)(498) + (591)(574) = 1{,}284{,}977$$

$$X_4X_4 = \sum_{c=1}^{n} X_{4c}X_{4c} = (342)^2 + (312)^2 + (398)^2 + (342)^2 + (431)^2 = 675{,}437$$

$$X_4X_5 = \sum_{c=1}^{n} X_{4c}X_{5c} = (342)(597) + (312)(455) + (398)(550) + (342)(498) + (431)(574) = 982{,}774$$

$$X_5X_5 = \sum_{c=1}^{n} X_{5c}X_{5c} = (597)^2 + (455)^2 + (550)^2 + (498)^2 + (574)^2 = 1{,}443{,}414$$

Figure 4. Sample Calculations Involved in the Development of the Cross Product Matrices (Continued)

Operator K, Shot Number 4
Five Element Breakdown

Cross Product Matrix

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
|---|---|---|---|---|---|
| $X_1$ | 395,171 | 773,360 | 686,535 | 513,244 | 751,389 |
| $X_2$ | | 1,595,447 | 1,326,121 | 985,846 | 1,465,750 |
| $X_3$ | | | 1,250,139 | 889,927 | 1,284,977 |
| $X_4$ | | | | 675,437 | 982,744 |
| $X_5$ | | | | | 1,443,441 |

Figure 4. Sample Calculations Involved in the Development of the Cross Product Matrices (Continued)

Operator K, Shot Number 4
Five Element Breakdown

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Calculation of Entries in Covariance Matrix (Refer to Figure 4. for Corresponding Entries in the Cross Product Matrix and the Sums for Each Element.)

$$X_1X_1 = \frac{n\sum\limits_{c=1}^{n} X_{1c}X_{1c} - \sum\limits_{c=1}^{n} X_{1c}\sum\limits_{c=1}^{n} X_{1c}}{n} = \frac{5(394{,}171) - (1405)(1405)}{5} = +366$$

$$X_1X_2 = \frac{5(773{,}360) - (1405)(2763)}{5} = -3{,}043$$

$$X_1X_3 = \frac{5(686{,}535) - (1405)(2437)}{5} = +1{,}738$$

$$X_1X_4 = \frac{5(513{,}244) - (1405)(1825)}{5} = +419$$

$$X_1X_5 = \frac{5(751{,}389) - (1405)(2674)}{5} = -5$$

$$X_2X_2 = \frac{5(1{,}595{,}447) - (2763)(2763)}{5} = +68{,}613$$

$$X_2X_3 = \frac{5(1{,}326{,}121) - (2763)(2437)}{5} = -20{,}565$$

$$X_2X_4 = \frac{5(985{,}846) - (2763)(1825)}{5} = -22{,}649$$

Figure 5. Sample Calculations Involved in the Development of the Covariance Matrices

Operator K, Shot Number 4
Five Element Breakdown

$$X_2X_5 = \frac{5(1,465,750) - (2763)(2674)}{5} = -11,902$$

$$X_3X_3 = \frac{5(1,250,139) - (2437)(2437)}{5} = +62,345$$

$$X_3X_4 = \frac{5(889,927) - (2437)(1825)}{5} = +422$$

$$X_3X_5 = \frac{5(1,284,977) - (2437)(2674)}{5} = -18,331$$

$$X_4X_4 = \frac{5(675,437) - (1825)(1825)}{5} = +9,312$$

$$X_4X_5 = \frac{5(982,744) - (1825)(2674)}{5} = +6,734$$

$$X_5X_5 = \frac{5(1,443,441) - (2674)(2674)}{5} = +13,359$$

Figure 5. Sample Calculations Involved in the Development of the Covariance Matrices (Continued)

Operator K, Shot Number 4
Five Element Breakdown

Covariance Matrix

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
|---|---|---|---|---|---|
| $X_1$ | 366 | -3,043 | 1,738 | 419 | -5 |
| $X_2$ | | 68,613 | -20,565 | -22,649 | -11,902 |
| $X_3$ | | | 62,345 | 422 | -18,331 |
| $X_4$ | | | | 9,312 | 6,734 |
| $X_5$ | | | | | 13,359 |

Figure 5. Sample Calculations Involved in the Development of the Covariance Matrices (Continued)

Operator K, Shot Number 4
Five Element Breakdown

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Evaluation of Numerator Covariance Matrix (Refer to Figure 5 for Corresponding Entries in the Covariance Matrix)

$$X_1^*X_1 = \sqrt{X_1X_1} = \sqrt{366} = 19.13$$

$$X_1^*X_2 = \frac{X_1X_2}{X_1^*X_2} = \frac{-3043}{19.13} = -159.07$$

$$X_1^*X_3 = \frac{X_1X_3}{X_1^*X_2} = \frac{1738}{19.13} = 90.85$$

$$X_1^*X_4 = \frac{X_1X_4}{X_1^*X_4} = \frac{419}{19.13} = 21.90$$

$$X_1^*X_5 = \frac{X_1X_5}{X_1^*X_5} = \frac{-5}{19.13} = -.26$$

$$X_2^*X_2 = \sqrt{X_2X_2 - (X_1^*X_2)^2} = \sqrt{68,613 - (-159.07)^2} = 208.11$$

$$X_2^*X_3 = \frac{X_2X_3 - (X_1^*X_2)(X_1^*X_3)}{X_2^*X_2} = \frac{-20,565 - (-159.07)(90.85)}{208.11} = -29.37$$

Figure 7. Sample Calculations Involved in the Computation of the L and L* Values

Operator K, Shot Number 4
Five Element Breakdown

$$X_2^*X_4 = \frac{X_2X_4 - (X_1^*X_2)(X_1^*X_4)}{X_2^*X_2} = \frac{-22,649 - (-159.07)(21.90)}{208.11} = -92.09$$

$$X_2^*X_5 = \frac{X_2X_5 - (X_1^*X_2)(X_1^*X_5)}{X_2^*X_2} = \frac{-11,902 - (-159.07)(-.26)}{208.11} = -57.39$$

$$X_3^*X_3 = \sqrt{X_3X_3 - (X_1^*X_3)^2 - (X_2^*X_3)^2} = \sqrt{62,345 - (90.85)^2 - (29.37)^2} = 230.71$$

$$X_3^*X_4 = \frac{X_3X_4 - (X_1^*X_3)(X_1^*X_4) - (X_2^*X_3)(X_2^*X_4)}{X_3^*X_3} = \frac{422 - (90.85)(21.90) - (-29.37)(-92.09)}{230.71} = -18.52$$

$$X_3^*X_5 = \frac{X_3X_5(X_1^*X_3)(X_1^*X_5) - (X_2^*X_3)(X_2^*X_5)}{X_3^*X_3} = \frac{-18,331 - (90.85)(-.26) - (29.37)(-57.39)}{230.71} = -86.66$$

$$X_4^*X_4 = \sqrt{X_4X_4 - (X_1^*X_4)^2 - (X_2^*X_4)^2 - (X_3^*X_4)^2} = \sqrt{9,312 - (21.90)^2 - (92.09)^2 - (18.52)^2} = 3.00$$

$$X_4^*X_5 = \frac{X_4X_5 - (X_1^*X_4)(X_1^*X_5) - (X_2^*X_4)(X_2^*X_5) - (X_3^*X_4)(X_3^*X_5)}{X_4^*X_4} = \frac{6,734 - (21.90)(-.26) - (-92.09)(-57.39)}{3.00}$$

$$\frac{-(-18.52)(-86.66)}{3.00} = -50.00$$

$$X_5^*X_5 = \sqrt{X_5X_5 - (X_1^*X_5)^2 - (X_2^*X_5)^2 - (X_3^*X_5)^2 - (X_4^*X_5)^2} = \sqrt{13,359 - (.26)^2 - (57.39)^2 - (86.66)^2 - (50.00)^2}$$

$$= 7.42$$

Figure 7. Sample Calculations Involved in the Computation of the L and L* Values (Continued)

Operator K, Shot Number 4
Five Element Breakdown

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Determinant Value
of Numerator Matrix $= (X_1^{*}X_1 \cdot X_2^{*}X_2 \cdot X_3^{*}X_3 \cdot X_4^{*}X_4 \cdot X_5^{*}X_5)^2 = (19.13 \cdot 208.11 \cdot 230.71 \cdot 3.00 \cdot 7.42)^2$

$= 3.69362 \times 10^{14}$

Determinate Value
of Denominator Matrix $= (X_1X_1 \cdot X_2X_2 \cdot X_3X_3 \cdot X_4X_4 \cdot X_5X_5) = (366 \cdot 68{,}613 \cdot 62{,}345 \cdot 9{,}312 \cdot 13{,}359)$

$= 1{,}947{,}620 \times 10^{14}$

$$L = \frac{|S1|^{n/2}}{|S2|^{n/2}} = \frac{3.69362}{1947620}^{5/2} = 0.0000019^{5/2}$$

$\mathrm{Log}_{10} L = 5/2 \; \mathrm{Log}_{10} \; (0.0000019)$

$2 \; \mathrm{Log}_{10} L = 5 \; \mathrm{Log}_{10} \; (0.0000019)$

$2 \; \mathrm{Log}_e L = 5(2.3) \; \mathrm{Log}_{10} \; (0.0000019)$

$L^* = -2 \; \mathrm{Log}_e L = - \; 5(2.3)\mathrm{Log}_{10}(0.0000019)$

$L^* = 42.794$

Figure 7. Sample Calculations Involved in the Computation of the L and L* Values (Continued)

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